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Elastic scattering of electrons by metastable hydrogen atoms in the presence of a resonant laser field

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Abstract. Within the framework of the rotating wave approximation the elastic scattering of electrons by metastable 2s state of hydrogen atoms is studied in the presence of a resonant laser field. The frequency of the circularly polarized laser field is chosen to match the 2s−3p transition frequency in the hydrogen atom. Variation of the cross section with laser intensity (10^6-10^{11} W cm^{-2}) and with incident electron energy (50−150 eV) is investigated.

PACS. 34.80.Qb Laser-modified scattering

1 Introduction

During the last two decades a remarkable resurgence of interest has taken place in the study of electron-atom scattering processes in the presence of electromagnetic radiation field. Apart from their prospective applications in number of physical processes *e.g.* in heating of plasma by electromagnetic radiation, laser induced gas breakdown, the population of metastable states *etc.*, such processes provide a fundamental understanding of the three body problem and relative coupling between the radiation and the bound and free charged particles. Due to the increasing availability of the tunable and intense lasers in a wide frequency range, experiments on the laser-assisted collision process have become feasible and are in progress at several laboratories (Mason *et al.* [1], Wallbank *et al.* [2], and Luan *et al.* [3]).

Theoretically, the electron-hydrogen atom scattering in the presence of a laser field has been investigated for a variety of special cases by many workers using different theoretical treatments (Rahman and Faisal [4], Mathur [5], Mittleman [6], Prasad and Unnikrishnan [7], Byron and Joachain [8], Dubois *et al.* [9], Francken and Joachain [10], Bhattacharya *et al.* [11] and Cionga and Florescu [12]. Although the exact wavefunction of a free charged particle embedded in a laser field can be represented accurately by Volkov state (Keldysh [13]), the central problem resides with the adequate description of laser modified atomic states. For example the usual perturbation approach breaks down at near resonance condition due to the appearance of a series of divergences (Jetzke *et al.* [14], Francken *et al.* [15]). If the laser frequency matches an atomic resonance, a two-level model can be used adequately. Hahn and Hertel [16], Gazazian [17], Gersten and Mittleman [18], Cavaliere *et al.* [19], Mittleman [20], Pundir and Mathur [21], Unnikrishnan [22] and Purohit and Mathur [23] used the two-state model and the rotating-wave approximation to represent the bound state of atom in the presence of a resonant laser field.

In this paper we report a theoretical calculation using a two-level model and rotating-wave approximation (Delone and Krainov [24], Pundir and Mathur [21]) to study the elastic scattering of electrons by metastable 2s state of hydrogen atoms in the presence of a circularly polarized resonant laser field. The frequency of the laser field is chosen to match with the 2s−3p transition frequency of atomic hydrogen.

The term elastic scattering is used here to describe those collisions in which the atomic state remains unchanged, but the projectile electron energy may change due to the transfer of photons between the electron-atom system and the laser field [8]. If l photons are transferred then,

$$k_\mathrm{f}^2 = k_\mathrm{i}^2 + 2l\hbar\omega$$

where $\mathbf{k}_\mathrm{i}(k_\mathrm{i}^2/2)$ and $\mathbf{k}_\mathrm{f}(k_\mathrm{f}^2/2)$, denote respectively the wavevectors (energy) of the free electron before and after the scattering. $l = 0$ would correspond to pure elastic scattering in the presence of the laser field. Here we consider the elastic scattering process in which the atomic metastable state (2s) remains unchanged and a single photon ($l = 1$) is exchanged between the laser field and the electron-atom system.

2 Theory

The S-matrix for the elastic scattering of electron by hydrogen atom in the presence of a resonant laser field is given, in the first order Born approximation of the electron atom scattering process, by,

$$S_{\pm} = -i \int < \psi_{\pm} \Phi_{\mathrm{f}} | U | \psi_{\pm} \phi_{\mathrm{i}} > \mathrm{d}t \quad (1)$$

where $\psi_{\pm}$ is the modified atomic state in the presence of a resonant laser field within the framework of the two-level model and the rotating-wave approximation (Delone and Krainov [24]). Subscript $\pm$ refer to the two different prescattering prescriptions of the target atom created due to the oscillation of the atomic electron between the initial and the resonant state. Φ_{i} and Φ_{f} are respectively the laser modified wavefunctions of the incident and the scattered electron given by the Volkov state [13]. The detailed wavefunctions $\psi_{\pm}, \Phi_{\mathrm{i}}, \Phi_{\mathrm{f}}$ used here are described in our earlier work [21]. U is the interaction potential between the projectile electron and the target hydrogen atom. Equation (1) would be valid when the electron energy is large compared with the excitation thresholds from the initial state. The range of electron energies studied here (50–150 eV) is more than fourteen times the excitation threshold for any excitation from metastable 2s state of hydrogen *via* electron impact in equation (1). At such high energies the use of the first Born approximation would be satisfactory. Further, the exchange effect, which is important at low energies, would be small in the energy range studied here and is therefore neglected in the present work.

Using the laser-atom interaction in the dipole approximation, the S_+ matrix is obtained as,

$$\begin{aligned} S_{+} = & -\mathrm{i} \sum_{n} \left[T_1^{+}(n) \delta \left(\frac{1}{2} k_{\mathrm{f}}^2 - \frac{1}{2} k_{\mathrm{i}}^2 + n\omega \right) \right. \\ & + T_2^{+}(n) \delta \left(\frac{1}{2} k_{\mathrm{f}}^2 - \frac{1}{2} k_{\mathrm{i}}^2 + n\omega - \omega \right) \\ & + T_3^{+}(n) \delta \left(\frac{1}{2} k_{\mathrm{f}}^2 - \frac{1}{2} k_{\mathrm{i}}^2 + n\omega + \omega \right) \\ & \left. + T_4^{+}(n) \delta \left(\frac{1}{2} k_{\mathrm{f}}^2 - \frac{1}{2} k_{\mathrm{i}}^2 + n\omega \right) \right] \end{aligned} \quad (2)$$

with

$$\begin{aligned} T_1^{+}(n) =& A_n < \mathrm{i} | \mathbf{O} | \mathrm{i} > \\ T_2^{+}(n) =& A_n \frac{V_{\mathrm{d}}^{*}}{(\eta + \Omega)} < \mathrm{i} | \mathbf{O} | \mathrm{r} > \\ T_3^{+}(n) =& A_n \frac{V_{\mathrm{d}}}{(\eta + \Omega)} < \mathrm{r} | \mathbf{O} | \mathrm{i} > \\ T_4^{+}(n) =& A_n \frac{V_{\mathrm{d}}^{*} V_{\mathrm{d}}}{(\eta + \Omega)^2} < \mathrm{r} | \mathbf{O} | \mathrm{r} > \end{aligned} \quad (3)$$

where

$$\begin{aligned} \mathbf{O} =& -\delta_{\mathrm{ir}} + \exp(i \mathbf{q} \cdot \mathbf{r}_1), \\ A_n =& \frac{8\pi^2}{q^2} C(\omega, E) C^{*}(\omega, E) i^n J_n(z) \\ & \times \exp(-iz \cos\phi_q) \exp(-in\phi_q), \\ z =& \frac{Eq \sin\theta_q}{\omega^2}, \\ \mathbf{q} =& \mathbf{k}_{\mathrm{i}} - \mathbf{k}_f. \end{aligned}$$

$J_n(z)$ is the Bessel function of the n^{th} order and $J_{-n}(z) = (-1)^n J_n(z)$ for integer n (Both $J_n(z)$ and $J_{-n}(z)$ are solutions of the Bessel differential equation). $\mathbf{q}$ is the momentum transfer vector with polar angles θ_q and ϕ_q.

$|i>$ and $|r>$ are the field-free initial (2s) and the resonant (3p) states of the target hydrogen atom with energies W_{i} and W_{r} respectively. η and Ω are the detuning parameter and the Rabi frequency respectively. $\omega(\hbar = 1)$ is the energy of the laser photon. The dipole coupling matrix element V_{d} is given by,

$$\begin{aligned} V_{\mathrm{d}} &= iE < \mathrm{i} | \hat{\boldsymbol{\epsilon}} \cdot \mathbf{r}_1 | \mathrm{r} > (W_{\mathrm{r}} - W_{\mathrm{i}})/\omega, \\ E &= (8\pi I / c_0)^{1/2} \end{aligned}$$

E is the laser electric field and I the laser intensity. c_0 is the velocity of light. $\hat{\boldsymbol{\epsilon}}$ is the unit polarisation vector of the laser field and $\mathbf{r}_1$ is the position vector of the valence electron. The coefficient $C(\omega, E)$ is defined as,

$$\begin{aligned} C(\omega, E) &= (\eta + \Omega) / [(\eta + \Omega)^2 + |V_{\mathrm{d}}|^2]^{1/2} \\ \eta &= \omega - (W_{\mathrm{r}} - W_{\mathrm{i}}), \ \Omega = (\eta^2 + |V_{\mathrm{d}}|^2)^{1/2}. \end{aligned}$$

The S_- matrix in equation (1) can also be evaluated in a similar way.

The first term in equation (2) describes the elastic scattering from the dressed 2s state in the presence of laser field with the exchange of one photon between the field and the projectile electron.

The second term describes the situation when one photon is absorbed by the atom from the radiation field raising it to the laser resonant atomic state, followed by superelastic scattering by electron impact from the laser excited resonant state.

The third term describes simultaneous raising of atom to resonant state by electron impact followed by the emission of photon from the resonant state.

The fourth term describes simultaneous raising of atom to resonant state by absorption of a photon, elastic electron scattering on the resonant state and emission of photon from the resonant state.

The differential cross sections (σ_+ and σ_-) for the elastic scattering of electrons by hydrogen atom in the presence of resonant laser field are given by,

$$\sigma_{+} = \frac{1}{4\pi^2} \frac{k_{\mathrm{f}}}{k_{\mathrm{i}}} |T_1^{+}(-1) + T_2^{+}(0) + T_3^{+}(-2) + T_4^{+}(-1)|^2 \quad (4)$$

$$\sigma_{-} = \frac{1}{4\pi^2} \frac{k_{\mathrm{f}}}{k_{\mathrm{i}}} |T_1^{-}(-1) + T_2^{-}(0) + T_3^{-}(-2) + T_4^{-}(-1)|^2, \quad (5)$$

and

$$\sigma = P_{+} \sigma_{+} + P_{-} \sigma_{-}. \quad (6)$$

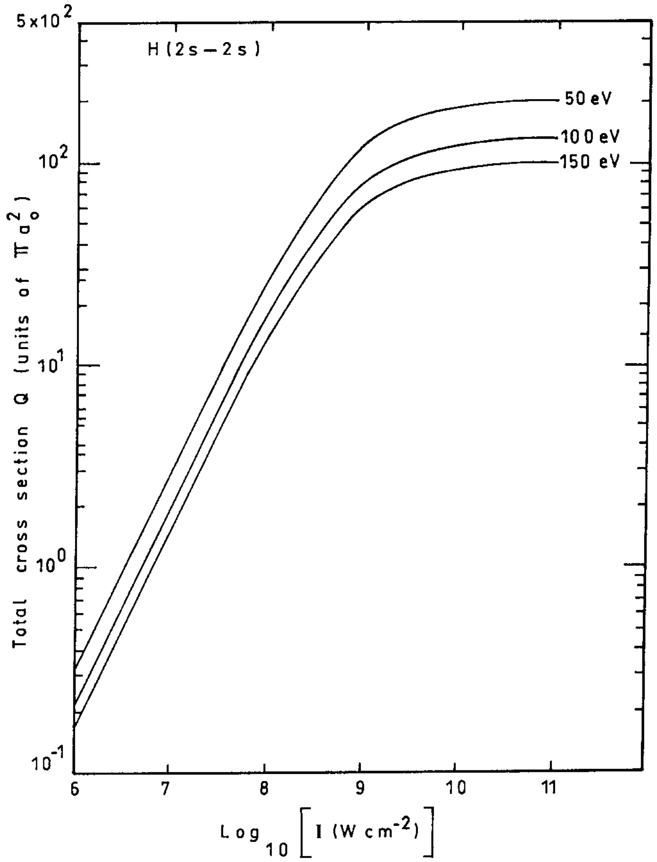

Fig. 1. Intensity variation of the total elastic cross section (Q) for the elastic scattering of metastable hydrogen atom. (——) present results in the presence of laser field.

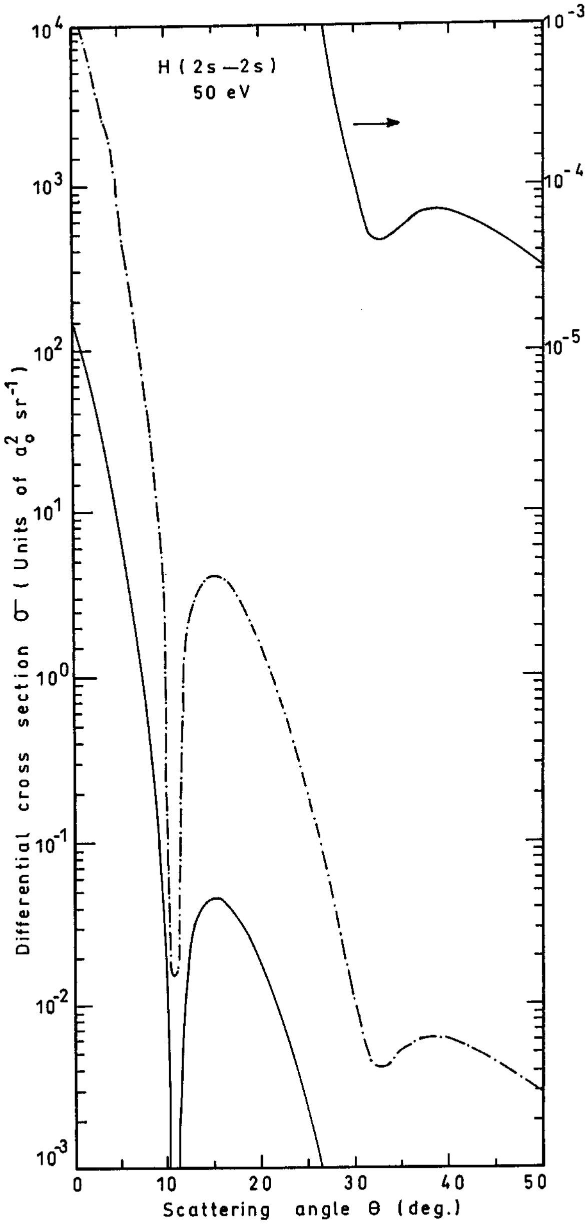

Fig. 2. The differential cross section (σ) for the elastic scattering of metastable hydrogen atom at 50 eV. (——) present result at laser intensity 10^6 W cm^{-2}; (— · — · —) present results at laser intensity 10^8 W cm^{-2}.

P$_+$ and P$_-$ are the respective probabilities of finding the atom in the state ψ_+ and ψ_- at the instant of collision reaction [21]. Since all the T matrices in equation (3) are the functions of the laser field strength E, the cross-sections in equations (4) and (5) will exhibit dependence on the laser intensity.

The total cross section Q is obtained as,

$$Q = \int \sigma \mathrm{d}\Omega' \tag{7}$$

3 Results and discussion

Our results for the variation of the total and the differential cross sections for the elastic scattering of electrons by metastable hydrogen atom in the presence of a resonant laser field are presented in Figures 1-2 respectively. We have considered the case of one photon exchange process. Results are obtained in the projectile energy range 50–150 eV and the laser intensity range of 10^6–10^{11} W cm^{-2}. The detuning parameter η is taken to be 10^{-4} a.u.

Figure 1 shows our results for the variation of the total cross-section (TCS) with the laser intensity (10^6–10^{11} W cm^{-2}) at 50, 100 and 150 eV incident electron energies. From the figure it is observed that the TCS increases rapidly in the range 10^6–10^9 Wcm^{-2} of laser intensity at all energies. Beyond this intensity range the increase in TCS is small and it tends to saturate. It may be pointed out that one can not make a proper comparison between the resonant field-assisted and field-free results, since in the former case the electron scattering proceeds from a mixture of two different prescattering prescriptions of the target atom while in the later case the scattering takes place from the pure initial state only. Thus there is no precise field-free counter part to the resonant laser field-assisted electron scattering.

Figure 2 shows the variation of the differential cross section (DCS) for the resonant laser-assisted elastic scattering of electrons by metastable hydrogen atoms at the incident electron energy of 50 eV and at two laser intensities 10^6 W cm^{-2} (continuous curve) and 10^8 W cm^{-2} (dash-dot curve). From the figure it is noticed that for both the laser intensities, the DCS first decreases to give a minima at small scattering angle (at about 11°) followed by a rapid increase giving a maxima at 15° scattering angle. Beyond this angle the DCS again decreases giving a dip at about 32° within the angular region shown in the figure. The relative magnitude of the DCS increases nearly by order of two in almost the entire angular region as the laser intensity changes from 10^6 to 10^8 W cm^{-2}.

From the above study it has been found that the resonant laser assisted cross sections are significantly large and therefore are measurable. It is also found that the laser effects are more prominent in the region of small scattering angles as compared to the large scattering angles. A significant increase in the cross sections with increasing laser intensity is similar to the findings in the case of laser-assisted electron impact excitation of atoms by Faisal and collaborators [25] and Mathur and collaborators [21, 23]. At present no experimental work is available to compare with our study. However, with the availability of laser of frequency corresponding to the (2s−3p) transition frequency (6563° A) in hydrogen (Cornet *et al.* [26]), it may become possible to perform experiments for the elastic and excitation (Purohit and Mathur [23]) processes in the scattering of electrons by metastable hydrogen atoms in the presence of a resonant laser field.

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